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Simulation of The Temperature Profile of The Curing Process of Thick Carbon Fiber Laminates Using The Age Algorithm

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ABSTRACT

This paper determines by simulation the temperature profiles of a thick carbon fiber/epoxy laminate during its cure process. The one-dimensional model of heat transfer is solved by using a finite difference numerical scheme. In the process, the same physical and thermal properties of materials used by ZhanSheng et al. are employed. The temperature is calculated through a judicious formulation and application of the Alternating Group Explicit (AGE) iterative method developed by Evans and Sahimi. A fourthorder Runge Kutta method is applied for the cure. The AGE scheme proves to be a viable iterative method with respect to stability, efficiency and rate of convergence.

Keywords: *Simulation of the temperature profile, age algorithm, composite*

1. INTRODUCTION

Bridge structures, tank and submarine hulls, and airplanes require the usage of thick composite laminates. It is important that the composite laminates are manufactured at low cost but of high quality. The improper manufacture of these thick composite laminates can lead to large thermal gradients and long processing times.

Several studies have been published on the curing of thick thermoset matrix composites. In their study, Zhan-Shen et al [5] obtained, by experiment, the temperature profiles of a thick carbon fiber/epoxy laminate during its cure process. By using the same physical and thermal properties of materials employed by the latter, this paper determines by simulation the temperature profiles of the laminate during its cure process. The one dimensional model of heat transfer is solved by using a finite difference numerical scheme. The temperature is calculated through a judicious formulation and application of the Alternating Group Explicit (AGE) iterative

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method developed by Evans and Sahimi [4]. A fourth-order Runge Kutta method is applied for the cure[1][2][3].

2. RESEARCH METHOD

2.1 Thermo-Chemical Model

A one-dimensional model of heat transfer is used to simulate the curing process of a 2 cm thick carbon fiber/epoxy laminate. It is assumed that the convective heat transfer effect by the resin flow is negligible and the resin and fiber are at the same temperature at any specific time.

The one-dimensional model of heat transfer is given by:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \rho H_\mu \frac{d\alpha}{dt}. \quad \text{(i)}$$

where $\frac{d\alpha}{dt} = K(T)\alpha^m (1-\alpha)^n$ and (ii)

$$K(T) = A\exp(-C_2 T), \quad \text{(iii)}$$

$$m = C_1 \exp(-C_2 T), \quad \text{(iv)}$$

$$n = C_3 \exp(-C_4 T). \quad \text{(v)}$$

where T is temperature, r, Cp and k are density, specific heat, and thermal conductivity of composite, respectively;dα/dt is cure rate; Hm is heat of reaction generated during dynamic scanning; A is preexponential factor; E is activation energy; R is universal gas constant;C1 ,C2,C3 and C4 are constant, respectively. The cure kinetic parameters of the carbon/epoxy composites, are presented in Table 1.

The degree of cure is temperature as well as spatially dependent. The initial temperature is known and is taken to be uniform. The initial degree of cure is α = 0, and theoretically reaches a value of 1 when the composite is fully cured. A fourth-order Runge Kutta was used to compute the degree of cure.

Table 1 Physical and Thermal Properties of The Used Material

Symbol	Properties	Value
AA	Pre-exponential factor	2.263e7/min.
EE H_u	Activation energy Heat of reaction	5.682e4 J/mol 313.84 J/g
RR	Universal gas constant	8.314472 J/Kmol
Rho-r	Density of resin	1.25 g/cm^3
Rho-f	Density of fiber	1.80 g/cm^3
C_R	Specific heat of resin	1.260 J/gK
C_f	Specific heat of fiber	0.712 J/gK
K_r	Conductivity of resin	0.24 W/mK =0.144J/(K min cm)
K_f	Conductivity of fiber	2.51 W/mK =1.506 J/(K min cm)
L	Length of composite	20 mm=2.0 cm

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2.2Experiment

In the experiment conducted by Zhan-Sheng et al, aT300/HD03 carbon epoxy prepreg was used. (Figure 1) .

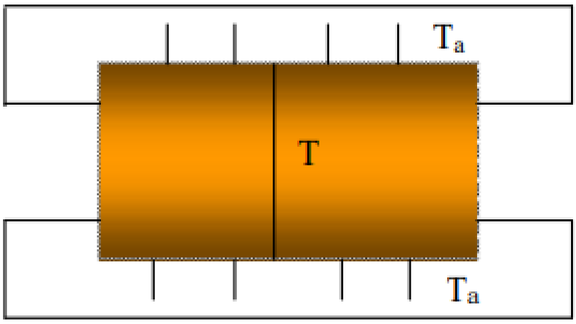

Figure 1 The Prepreg

During the experiment, the autoclave temperature is initially increased to1300°C with the rate of 2 °C/min and kept at this temperature for 60 min. At this stage, consolidation takes place. The excess resin is squeezed out of the laminate. Then, the temperature is increased to 1700°C with the same rate of 2 °C /min. The temperature is maintained for 180 min to complete the cure. Physical and thermal properties of the used materials are shown in Table 2

Table 2. Cure Kinetic Parameters of The Carbon/Epoxy Composite

Symbol	Value	Symbol	Value
C_1	1.879e-10	C_3	1.94e-3
C_2	6.06e-2	C_4	-1.49e-2

The simple rule of mixture was employed for the physical and thermal properties of composite such as density (ρ), specific heat (Cp), and conductivity (k) [5].

$$k = V_f K_f + (1 - V_f) K_r$$

$$C_P = V_f C_f + (1 - V_f) C_r$$

$$\rho = V_f \rho_f + (1 - V_f) \rho_r$$

where K_f, K_r, C_f, C_r, ρ_f, ρ_r are conductivity of fiber, conductivity of resin, specific heat of fiber, specific heat of resin, density of fiber and density of resin respectively.

3.1 Solution Techniques

The initial temperature of the composite is labeled as T_i, and the applied temperature at the top and the bottom of the composite is labeled as T_aThe thickness of the composite is taken as L. To non-dimensionalise the model, we introduce the variables t*, T* dan z* where

$$z^* = \frac{z}{L} \qquad (1)$$

So that z = z* L (2)

$$T^* = \frac{T - T_i}{T_a - T_i} \qquad (3)$$

so that $T = T^* (T_a - T_i) + T_i$ (4)

$$t^* = t\left(\frac{k}{\rho \, C_p L^2}\right) \qquad (5)$$

so that $t = t^{*}\left(\frac{\rho\ C_p L^2}{k}\right)$ (6)

From (4),

$$\frac{\partial T}{\partial t^{*}} = \frac{\partial T^{*}}{\partial t^{*}}(T_a - T_i) + \frac{\partial T_i}{\partial t^{*}} = \frac{\partial T^{*}}{\partial t^{*}}(T_a - T_i) \quad (7)$$

$$\frac{\partial T}{\partial z^{*}} = \frac{\partial T^{*}}{\partial z^{*}}(T_a - T_i) + \frac{\partial T_i}{\partial z^{*}} = \frac{\partial T^{*}}{\partial z^{*}}(T_a - T_i) \quad (8)$$

$$\frac{\partial^2 T}{\partial z^{*2}} = \frac{\partial^2 T^{*}}{\partial z^{*2}}(T_a - T_i) \quad (9)$$

By the Chain Rule,

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial t^{*}} \cdot \frac{\partial t^{*}}{\partial t}$$

$$= \frac{\partial T}{\partial t^{*}} \cdot \left(\frac{k}{\rho\ C_p L^2}\right) \quad \text{(from (5))}$$

$$= \frac{\partial T^{*}}{\partial t^{*}} \cdot (T_a - T_i)\left(\frac{k}{\rho\ C_p L^2}\right) \quad \text{(from (7))}$$

$$\frac{\partial T}{\partial t} = \frac{\partial T^{*}}{\partial t^{*}}\left(\frac{k(T_a - T_i)}{\rho\ C_p L^2}\right) \quad (10)$$

By the Chain Rule,

$$\frac{\partial T}{\partial z} = \frac{\partial T}{\partial z^{*}} \cdot \frac{\partial z}{\partial z}$$

$$= \frac{\partial T}{\partial z^{*}} \cdot \left(\frac{1}{L}\right) \quad \text{(from(1))} \quad (11)$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) = \frac{\partial}{\partial z^{*}}\left(\frac{\partial T}{\partial z}\right)\frac{\partial z^{*}}{\partial z}$$

$$= \frac{\partial}{\partial z^{*}}\left(\frac{\partial T}{\partial z}\right)\frac{1}{L} \quad \text{(from (1))}$$

$$= \frac{\partial}{\partial z^{*}}\left(\frac{\partial T}{\partial z^{*}}\right)\left(\frac{1}{L}\right)\left(\frac{1}{L}\right) = \frac{\partial^2 T^{*}}{\partial z^{*2}}\left(\frac{1}{L^2}\right)$$

$$= \frac{\partial^2 T^{*}}{\partial z^{*2}}\left(\frac{T_a - T_i}{L^2}\right) \quad \text{(from (9))} \quad (12)$$

By the Chain Rule,

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$$\frac{d\alpha}{dt} = \frac{d\alpha}{dt^*} \cdot \frac{dt^*}{dt}$$

$$= \frac{d\alpha}{dt^*} \cdot \frac{k}{\rho\, C_p L^2} \quad \text{(from (5))} \qquad (13)$$

$$\Rightarrow \frac{d\alpha}{dt^*} = \frac{\rho\, C_p L^2}{k} \frac{d\alpha}{dt} \qquad (14)$$

The model, in (i):

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \rho H_u \frac{d\alpha}{dt}$$

$$= k \frac{\partial^2 T}{\partial z^2} + \rho H_u \frac{d\alpha}{dt}$$

From (10), (12) and (13),

$$\rho C_p \left(\frac{k(T_a - T_i)}{\rho C_p L^2}\right) \frac{\partial T^*}{\partial t^*} = k\left(\frac{T_a - T_i}{L^2}\right) \frac{\partial^2 T^*}{\partial z^{*2}} +$$

$$\rho H_u \left(\frac{k}{\rho C_p L^2}\right) \frac{d\alpha}{dt^*}$$

$$\frac{k}{L^2}(T_a - T_i) \frac{\partial T^*}{\partial t^*} = \frac{k}{L^2}\left(\frac{T_a - T_i}{}\right) \frac{\partial^2 T^*}{\partial z^{*2}} +$$

$$\frac{H_u k}{C_p L^2} \frac{d\alpha}{dt^*}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial z^{*2}} + \frac{H_u k}{C_p L^2} \cdot \frac{L^2}{k(T_a - T_i)} \frac{d\alpha}{dt^*}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial z^{*2}} + \frac{H_u}{C_p (T_a - T_i)} \frac{d\alpha}{dt^*}$$

Let $N = \dfrac{H_u}{C_P (T_a - T_i)}$.

Then $\dfrac{\partial T^*}{\partial t^*} = \dfrac{\partial^2 T^*}{\partial z^{*2}} + N \dfrac{d\alpha}{dt^*}$.

Let $N \dfrac{d\alpha}{dt^*} = \beta_{ij}$.

$$\text{Hence } \frac{\partial T^*}{\partial t^*} = \frac{\partial^2 T^*}{\partial z^{*2}} + \beta_{ij} \,. \qquad (15)$$

Equation (15) is the non-dimensionalised model.
For clarity, let us relabel * T by T , * t by t, and * z by z . Then (15) becomes:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial z^2} + \beta_{ij} \,.$$

3.2 AGE Method

The temperature profiles in the laminate during the cure process can be obtained by applying the Alternating Group Explicit (AGE) finite difference iterative method developed by Evans and Sahimi. This iterative method employs the fractional splitting strategy which is applied alternately at each half (intermediate) time step on tridiagonal systems of difference schemes and which has proved to be stable. A weighted approximation (15) is given by:

$$\frac{T_{i,j+1}-T_{i,j}}{\Delta t}=\frac{1}{(\Delta z)^2}\begin{bmatrix}\theta(T_{i+1,j+1}-2T_{i,j+1}+T_{i-1,j+1})\\+(1-\theta)(T_{i+1,j}-2T_{i,j}+T_{i-1,j})\end{bmatrix}$$
$$+\beta_{ij}$$

Δt is the time step. The real line $0 \le z \le 1$ is divided into subintervals of length Δz. The parameter θ is an adjustable parameter varying between 0 and 1, and the algorithm depends on the chosen value of θ.

$$T_{i,j+1}-T_{i,j}=\frac{\Delta t}{(\Delta z)^2}\begin{bmatrix}\theta(T_{i+1,j+1}-2T_{i,j+1}+T_{i-1,j+1})\\+(1-\theta)(T_{i+1,j}-2T_{i,j}+T_{i-1,j})\end{bmatrix}$$
$$+\beta_{ij}\Delta t$$

Let $\lambda=\frac{\Delta t}{(\Delta z)^2}$, the mesh ratio.

Then

$$T_{i,j+1}-T_{i,j}=\lambda\begin{bmatrix}\theta(T_{i+1,j+1}-2T_{i,j+1}+T_{i-1,j+1})\\+(1-\theta)(T_{i+1,j}-2T_{i,j}+T_{i-1,j})\end{bmatrix}$$
$$+\beta_{ij}\Delta t$$

$$T_{i,j+1}-\lambda\theta T_{i+1,j+1}+2\lambda\theta T_{i,j+1}-\lambda\theta T_{i-1,j+1}=$$
$$T_{i,j}+\lambda(1-\theta)T_{i+1,j}-2\lambda(1-\theta)T_{i,j}+$$
$$\lambda(1-\theta)T_{i-1,j}+\beta_{ij}\Delta t$$

$$-\lambda\theta T_{i+1,j+1}+(1+2\lambda\theta)T_{i,j+1}-\lambda\theta T_{i-1,j+1}=$$
$$\lambda(1-\theta)T_{i+1,j}+[1-2\lambda(1-\theta)]T_{i,j}+$$
$$\lambda(1-\theta)T_{i-1,j}+\beta_{ij}\Delta t$$

Let $a=1+2\lambda\theta$, $b=-\lambda\theta$, and $c=-\lambda\theta$.

For $i=1$,

$$-\lambda\theta T_{2,j+1}+(1+2\lambda\theta)T_{1,j+1}-\lambda\theta T_{0,j+1}=$$
$$\lambda(1-\theta)T_{2,j}+[1-2\lambda(1-\theta)]T_{1,j}+\lambda(1-\theta)T_{0,j}$$
$$+\beta_{1j}\Delta t$$

$$-\lambda\theta T_{2,j+1}+(1+2\lambda\theta)T_{1,j+1}=\lambda(1-\theta)[T_{2,j}+T_{0,j}]$$
$$+[1-2\lambda(1-\theta)]T_{1,j}+\lambda\theta T_{0,j+1}+\beta_{1j}\Delta t$$

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Let

$$f_1 = \lambda(1-\theta)[T_{0,j} + T_{2,j}] + \lambda\theta T_{0,j+1} + [1-2\lambda(1-\theta)]T_{1,j} + \beta_{1j}\Delta t$$

Hence $aT_1 + bT_2 = f_1$.

For $i = 2,3,\ldots,m-1$,

$$-\lambda\theta T_{i+1,j+1} + (1+2\lambda\theta)T_{i,j+1} - \lambda\theta T_{i-1,j+1} = \lambda(1-\theta)T_{i+1,j} + [1-2\lambda(1-\theta)]T_{i,j} + \lambda(1-\theta)T_{i-1,j} + \beta_{ij}\Delta t$$

Let

$$f_i = \lambda(1-\theta)[T_{i-1,j} + T_{i+1,j}] + [1-2\lambda(1-\theta)]T_{i,j} + \beta_{ij}\Delta t.$$

Hence $bT_{i+1} + aT_i + cT_{i-1} = f_i$.

For $i = m$,

$$-\lambda\theta T_{m+1,j+1} + (1+2\lambda\theta)T_{m,j+1} - \lambda\theta T_{m-1,j+1} = \lambda(1-\theta)T_{m+1,j} + [1-2\lambda(1-\theta)]T_{m,j} + \lambda(1-\theta)T_{m-1,j} + \beta_{mj}\Delta t$$

$$(1+2\lambda\theta)T_{m,j+1} - \lambda\theta T_{m-1,j+1} = \lambda(1-\theta)[T_{m+1,j} + T_{m-1,j}] + [1-2\lambda(1-\theta)]T_{m,j} + \lambda\theta T_{m+1,j+1} + \beta_{mj}\Delta t$$

Let

$$f_m = \lambda(1-\theta)[T_{m+1,j} + T_{m-1,j}] + [1-2\lambda(1-\theta)]T_{m,j} + \lambda\theta T_{m+1,j+1} + \beta_{mj}\Delta t$$

Hence $aT_m + cT_{m-1} = f_m$.

Define $a = 1+2\lambda\theta$, $b = -\lambda\theta$, and $c = -\lambda\theta$ and

$$f_1 = \lambda(1-\theta)[T_{0,j} + T_{2,j}] + [1-2\lambda(1-\theta)]T_{1,j} + \lambda\theta T_{0,j+1} + \beta_{1j}\Delta t,$$

$$f_i = \lambda(1-\theta)[T_{i-1,j} + T_{i+1,j}] + [1-2\lambda(1-\theta)]T_{i,j} + \beta_{ij}\Delta t, i = 2,3,\ldots,m-1.$$

$$f_m = \lambda(1-\theta)[T_{m-1,j} + T_{m+1,j}] + [1-2\lambda(1-\theta)]T_{m,j} + \lambda\theta T_{m+1,j+1} + \beta_{mj}\Delta t$$

Hence $aT_m + cT_{m-1} = f_m$.

Define $a = 1 + 2\lambda\theta$, $b = -\lambda\theta$, and $c = -\lambda\theta$

and

$$f_1 = \lambda(1-\theta)[T_{0,j} + T_{2,j}] + [1 - 2\lambda(1-\theta)]T_{1,j} + \lambda\theta T_{0,j+1} + \beta_{1j}\Delta t,$$

$$f_i = \lambda(1-\theta)[T_{i-1,j} + T_{i+1,j}] + [1 - 2\lambda(1-\theta)]T_{i,j} + \beta_{ij}\Delta t, i = 2,3,\ldots, m-1.$$

$$f_m = \lambda(1-\theta)[T_{m-1,j} + T_{m+1,j}] + [1 - 2\lambda(1-\theta)]T_{m,j} + \lambda\theta T_{m+1,j+1} + \beta_{mj}\Delta t$$

we have

$$aT_1 + bT_2 = f_1 \quad (16)$$

$$cT_{i-1} + aT_i + bT_{i+1} = f_i \quad (17)$$

$$cT_{m-1} + aT_m = f_m \quad (18)$$

we have

$$aT_1 + bT_2 = f_1 \quad (16)$$

$$cT_{i-1} + aT_i + bT_{i+1} = f_i \quad (17)$$

$$cT_{m-1} + aT_m = f_m \quad (18)$$

Equations (16) through (18) can be displayed in a more compact matrix form as

$$\begin{bmatrix} a & b & & & & & \\ c & a & b & & \mathbf{0} & & \\ 0 & c & a & b & & & \\ & & \ddots & \ddots & \ddots & & \\ & \mathbf{0} & & \ddots & \ddots & \ddots & \\ & & & & c & a & b \\ & & & & & c & a \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ . \\ . \\ . \\ T_{m-1} \\ T_m \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ . \\ . \\ . \\ f_{m-1} \\ f_m \end{bmatrix}$$

or,

$$Au = f$$

where A is an mxm matrix, and u and f are mx1 matrices. Let us assume that we have an even number of intervals (corresponding to an odd number of internal points, i.e., m odd) on the real line $0 \leq z \leq 1$. We can then perform the following splitting of the coefficient matrix A: $A = G_1 + G_2$

where

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$$G_1 = \begin{bmatrix} \frac{a}{2} & & & & & \\ & \begin{matrix} \frac{a}{2} & b \\ c & \frac{a}{2} \end{matrix} & & & & \\ & & \begin{matrix} \frac{a}{2} & b \\ c & \frac{a}{2} \end{matrix} & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & & \ddots & \\ & & & & & \begin{matrix} \frac{a}{2} & b \\ c & \frac{a}{2} \end{matrix} \end{bmatrix}_{(m\times m)}$$

and

$$G_2 = \begin{bmatrix} \begin{matrix} \frac{a}{2} & b \\ c & \frac{a}{2} \end{matrix} & & & & & \\ & \begin{matrix} \frac{a}{2} & b \\ c & \frac{a}{2} \end{matrix} & & & & \\ & & \ddots & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & & \begin{matrix} \frac{a}{2} & b \\ c & \frac{a}{2} \end{matrix} & \\ & & & & & \frac{a}{2} \end{bmatrix}_{(m\times m)}$$

Relabel T by u . The AGE method using the Peaceman and Rachford variant for the stationary case (where r is a constant) is given by

$$(G_1 + rI)\mathbf{u}^{(p+\frac{1}{2})} = (rI - G_2)\mathbf{u}^{(p)} + \mathbf{f}\,,$$

$$(G_2 + rI)\mathbf{u}^{(p+1)} = (rI - G_1)\mathbf{u}^{(p+\frac{1}{2})} + \mathbf{f}\,,\ p \geq 0$$

or explicitly by,

$$\mathbf{u}^{(p+\frac{1}{2})} = (G_1 + rI)^{-1}[(rI - G_2)\mathbf{u}^{(p)} + \mathbf{f}] \qquad (19)$$

$$\mathbf{u}^{(p+1)} = (G_2 + rI)^{-1}[(rI - G_1)\mathbf{u}^{(p+\frac{1}{2})} + \mathbf{f}] \qquad (20)$$

In (19),

$$rI - G_2 = \begin{bmatrix} r & & & & & & & \\ & r & & & & & & \\ & & r & & & \mathbf{0} & & \\ & & & \ddots & & & & \\ & & \mathbf{0} & & \ddots & & & \\ & & & & & r & & \\ & & & & & & r & \\ & & & & & & & r \end{bmatrix}$$

$$-\left[\begin{array}{cc|cc|c|c|cc|c}
\frac{a}{2} & b & & & & & & & \\
c & \frac{a}{2} & & & & & & & \\
\hline
 & & \frac{a}{2} & b & & & & & \\
 & & c & \frac{a}{2} & & & & & \\
\hline
 & & & & \ddots & & \mathbf{0} & & \\
\hline
 & & & & & \ddots & & & \\
\hline
 & & \mathbf{0} & & & & \frac{a}{2} & b & \\
 & & & & & & c & \frac{a}{2} & \\
\hline
 & & & & & & & & \frac{a}{2}
\end{array}\right]_{(m\times m)}$$

$$=\left[\begin{array}{cc|cc|c|c|cc|c}
r-\frac{a}{2} & -b & & & & & & & \\
-c & r-\frac{a}{2} & & & & & & & \\
\hline
 & & r-\frac{a}{2} & -b & & & & & \\
 & & -c & r-\frac{a}{2} & & & & & \\
\hline
 & & & & \ddots & & \mathbf{0} & & \\
\hline
 & & & & & \ddots & & & \\
\hline
 & & \mathbf{0} & & & & r-\frac{a}{2} & -b & \\
 & & & & & & -c & r-\frac{a}{2} & \\
\hline
 & & & & & & & & r-\frac{a}{2}
\end{array}\right]_{(m\times m)}$$

Let $r_1 = r - \frac{a}{2}.$ Then

$$rI - G_2 =
\left[\begin{array}{cc|cc|c|c|cc|c}
r_1 & -b & & & & & & & \\
-c & r_1 & & & & & & & \\
\hline
 & & r_1 & -b & & & & & \\
 & & -c & r_1 & & & & & \\
\hline
 & & & & \ddots & & \mathbf{0} & & \\
\hline
 & & & & & \ddots & & & \\
\hline
 & & \mathbf{0} & & & & r_1 & -b & \\
 & & & & & & -c & r_1 & \\
\hline
 & & & & & & & & r_1
\end{array}\right]_{(m\times m)}$$

Hence

$$(rI - G_2)\mathbf{u}^{(p)} + \mathbf{f} =
\left[\begin{array}{cc|cc|c|c|cc|c}
r_1 & -b & & & & & & & \\
-c & r_1 & & & & & & & \\
\hline
 & & r_1 & -b & & & & & \\
 & & -c & r_1 & & & & & \\
\hline
 & & & & \ddots & & \mathbf{0} & & \\
\hline
 & & & & & \ddots & & & \\
\hline
 & & \mathbf{0} & & & & r_1 & -b & \\
 & & & & & & -c & r_1 & \\
\hline
 & & & & & & & & r_1
\end{array}\right]
\begin{bmatrix} u_1^{(p)} \\ u_2^{(p)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_m^{(p)} \end{bmatrix}
+
\begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ f_m \end{bmatrix}$$

Similarly, in (20),

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$$rI - G_1 = \begin{bmatrix} r_1 & & & & & & & & \\ & r_1 & -b & & & & & & \\ & -c & r_1 & & & & & & \\ & & & r_1 & -b & & & \mathbf{0} & \\ & & & -c & r_1 & & & & \\ & & & & & \ddots & & & \\ & \mathbf{0} & & & & & \ddots & & \\ & & & & & & & r_1 & -b \\ & & & & & & & -c & r_1 \end{bmatrix}_{(m\times m)}$$

Hence,

$$(rI - G_1)\mathbf{u}^{(p+\frac{1}{2})} + \mathbf{f} = \begin{bmatrix} r_1 & & & & & & & & \\ & r_1 & -b & & & & & & \\ & -c & r_1 & & & & & & \\ & & & r_1 & -b & & & \mathbf{0} & \\ & & & -c & r_1 & & & & \\ & & & & & \ddots & & & \\ & \mathbf{0} & & & & & \ddots & & \\ & & & & & & & r_1 & -b \\ & & & & & & & -c & r_1 \end{bmatrix} \begin{bmatrix} u_1^{(p+\frac{1}{2})} \\ u_2^{(p+\frac{1}{2})} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_m^{(p+\frac{1}{2})} \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ f_m \end{bmatrix}$$

Now

$$G_1 + rI = \begin{bmatrix} \frac{a}{2}+r & & & & & & & & \\ & \frac{a}{2}+r & b & & & & & & \\ & c & \frac{a}{2}+r & & & & & & \\ & & & \frac{a}{2}+r & b & & & \mathbf{0} & \\ & & & c & \frac{a}{2}+r & & & & \\ & & & & & \ddots & & & \\ & \mathbf{0} & & & & & \ddots & & \\ & & & & & & & \frac{a}{2}+r & b \\ & & & & & & & c & \frac{a}{2}+r \end{bmatrix}_{(m\times m)}$$

and

$$G_2 + rI = \begin{bmatrix} \frac{a}{2}+r & b & & & & & & & \\ c & \frac{a}{2}+r & & & & & & & \\ & & \frac{a}{2}+r & b & & & & & \\ & & c & \frac{a}{2}+r & & & & & \\ & & & & \ddots & & & \mathbf{0} & \\ & & & & & \ddots & & & \\ & \mathbf{0} & & & & & \frac{a}{2}+r & b & \\ & & & & & & c & \frac{a}{2}+r & \\ & & & & & & & & \frac{a}{2}+r \end{bmatrix}_{(m\times m)}$$

Let $\hat{G} = \begin{bmatrix} \frac{a}{2}+r & b \\ c & \frac{a}{2}+r \end{bmatrix} = \begin{bmatrix} r_2 & b \\ c & r_2 \end{bmatrix}$ where $r_2 = \frac{a}{2} + r$.

Hence,

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$$G_1 + rI = \begin{bmatrix} r_2 & & & & & \\ & \hat{G} & & & & \\ & & \hat{G} & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & & \ddots & \\ & & & & & \hat{G} \end{bmatrix}_{(m\times m)}$$

and

$$G_2 + rI = \begin{bmatrix} \hat{G} & & & & & \\ & \hat{G} & & & & \\ & & \hat{G} & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & & \ddots & \\ & & & & & r_2 \end{bmatrix}_{(m\times m)}$$

We see that G + rI 1 and G + rI 2 are block diagonal matrices. Therefore, G + rI 1 and G + rI 2 can be easily inverted. We have

$$(G_1 + rI)^{-1} = \begin{bmatrix} \frac{1}{r_2} & & & & & \\ & \hat{G}^{-1} & & & & \\ & & \hat{G}^{-1} & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & & \ddots & \\ & & & & & \hat{G}^{-1} \end{bmatrix}_{(m\times m)}$$

and

$$(G_2 + rI)^{-1} = \begin{bmatrix} \hat{G} & & & & & \\ & \hat{G}^{-1} & & & & \\ & & \hat{G}^{-1} & & \mathbf{0} & \\ & & & \ddots & & \\ & \mathbf{0} & & & \ddots & \\ & & & & & \frac{1}{r_2} \end{bmatrix}_{(m\times m)}$$

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$$\hat{G}^{-1} = \begin{bmatrix} r_2 & -b \\ -c & r_2 \end{bmatrix} \cdot \frac{1}{r_2^{\,2} - bc}$$

$$= \frac{1}{\Delta}\begin{bmatrix} r_2 & -b \\ -c & r_2 \end{bmatrix}$$

But

where $\Delta = r_2^2 - bc$

Therefore

$$(G_1 + rI)^{-1} = \frac{1}{\Delta}\begin{bmatrix} \frac{\Delta}{r_2} & & & & & & & \\ & r_2 & -b & & & & & \\ & -c & r_2 & & & & & \\ & & & r_2 & -b & & \mathbf{0} & \\ & & & -c & r_2 & & & \\ & & & & & \ddots & & \\ & \mathbf{0} & & & & & \ddots & \\ & & & & & & & r_2 & -b \\ & & & & & & & -c & r_2 \end{bmatrix}_{(m \times m)}$$

And

$$(G_2 + rI)^{-1} = \frac{1}{\Delta}\begin{bmatrix} r_2 & -b & & & & & & \\ -c & r_2 & & & & & & \\ & & r_2 & -b & & & & \\ & & -c & r_2 & & & & \\ & & & & \ddots & & \mathbf{0} & \\ & & & & & \ddots & & \\ & \mathbf{0} & & & & & r_2 & -b & \\ & & & & & & -c & r_2 & \\ & & & & & & & & \frac{1}{r_2} \end{bmatrix}_{(m \times m)}$$

From (19),

$$\mathbf{u}^{(p+\frac{1}{2})} = (G_1 + rI)^{-1}[(rI - G_2)\mathbf{u}^{(p)} + \mathbf{f}].$$

In matrix form,

$$\begin{bmatrix} u_1^{(p+\frac{1}{2})} \\ u_2^{(p+\frac{1}{2})} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_m^{(p+\frac{1}{2})} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} \frac{\Delta}{r_2} & & & & & & & \\ & r_2 & -b & & & & & \\ & -c & r_2 & & & & & \\ & & & r_2 & -b & & \mathbf{0} & \\ & & & -c & r_2 & & & \\ & & & & & \ddots & & \\ & \mathbf{0} & & & & & \ddots & \\ & & & & & & & r_2 & -b \\ & & & & & & & -c & r_2 \end{bmatrix}$$

$$
\cdot\begin{bmatrix} r_1 u_1^{(p)} - b u_2^{(p)} + f_1 \\ -c u_1^{(p)} + r u_2^{(p)} + f_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ r_1 u_m^{(p)} + f_m \end{bmatrix}
$$

From (20),

$$
\mathbf{u}^{(p+1)} = (G_2 + rI)^{-1}[(rI - G_1)\mathbf{u}^{(p+\frac{1}{2})} + \mathbf{f}].
$$

In matrix form,

$$
\begin{bmatrix} u_1^{(p+1)} \\ u_2^{(p+1)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_m^{(p+1)} \end{bmatrix} = \frac{1}{\Delta}\left[\begin{array}{cc|cc|cc|c|c|c}
r_2 & -b & & & & & & & \\
-c & r_2 & & & & & & & \\ \hline
 & & r_2 & -b & & & & & \\
 & & -c & r_2 & & & & & \\ \hline
 & & & & r_2 & -b & & \mathbf{0} & \\
 & & & & -c & r_2 & & & \\ \hline
 & & & & & & \ddots & & \\ \hline
 & & \mathbf{0} & & & & & \ddots & \\ \hline
 & & & & & & & & \frac{\Delta}{r_2}
\end{array}\right]
$$

$$
\cdot\begin{bmatrix} r_1 u_1^{(p+\frac{1}{2})} + f_1 \\ r_1 u_2^{(p+\frac{1}{2})} - b u_3^{(p+\frac{1}{2})} + f_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ -c u_{m-1}^{(p+\frac{1}{2})} + r_1 u_m^{(p+\frac{1}{2})} + f_m \end{bmatrix}
$$

The corresponding explicit expressions for the AGE equations are obtained by carrying out the multiplications in the last two equations above. Thus we have,

(i) at level $(p+\frac{1}{2})$:

$$
u_1^{(p+\frac{1}{2})} = \frac{1}{r_2}[r_1 u_1^{(p)} - b u_2^{(p)} + f_1]
$$

For $i = 2,3,\ldots,m-1.$

$$
u_i^{(p+\frac{1}{2})} = \frac{1}{\Delta}[A u_{i-1}^{(p)} + B u_i^{(p)} + C u_{i+1}^{(p)} + D u_{i+2}^{(p)} + E_i],
$$

and

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$$u_{i+1}^{(p+\frac{1}{2})} = \frac{1}{\Delta}[A u_{i-1}^{(p)} + B u_i^{(p)} + C u_{i+2}^{(p)} + \tilde{D} u_{i+2}^{(p)} + \tilde{E}_i]$$

where $A = -cr_2$, $B = r_1 r_2$, $C = -br_2$,

$$D = \begin{cases} 0, & i = m-1 \\ b^2, & otherwise \end{cases}, \quad E_i = r_2 f_i - bf_{i+1},$$

$$\tilde{A} = c^2, \ \tilde{B} = -cr_1, \ \tilde{C} = r_1 r_2,$$

$$\tilde{D} = \begin{cases} 0, & i = m-1 \\ -br_2, & otherwise \end{cases}, \quad \tilde{E}_i = -cf_i + r_2 f_{i+1}$$

(ii) at level $p+1$:

For $i = 1,3,\ldots,m-2$,

$$u_i^{(p+1)} = \frac{1}{\Delta}[P u_{i-1}^{(p+\frac{1}{2})} + Q u_i^{(p+\frac{1}{2})} + R u_{i+1}^{(p+\frac{1}{2})} + S u_{i+2}^{(p+\frac{1}{2})} + V_i]$$

and

$$u_{i+1}^{(p+1)} = \frac{1}{\Delta}[\tilde{P} u_{i-1}^{(p+\frac{1}{2})} + \tilde{Q} u_i^{(p+\frac{1}{2})} + \tilde{R} u_{i+1}^{(p+\frac{1}{2})} + \tilde{S} u_{i+2}^{(p+\frac{1}{2})} + \tilde{V}_i]$$

$$u_m^{(p+1)} = \frac{1}{r_2}[-cu_{m-1}^{(p+\frac{1}{2})} + r_1 u_m^{(p+\frac{1}{2})} + f_m]$$

where

$$P = \begin{cases} 0, & i = 1 \\ -cr_2, & i \neq 1 \end{cases}, \quad Q = r_1 r_2, \ R = -br_1,$$

$$S = b^2, \quad V_i = r_2 f_i - bf_{i+1},$$

$$\tilde{P} = \begin{cases} 0, & i = 1 \\ c^2, & i \neq 1 \end{cases}, \quad \tilde{Q} = -cr_1, \ \tilde{R} = r_1 r_2,$$

$$\tilde{S} = -br_2, \quad \tilde{V}_i = -cf_i + r_2 f_{i+1}$$

3. RESULT AND DISCUSSION

The simulation to obtain the temperature profiles during the cure process is performed through the vertical cross section of the laminate. In the process, heat is transferred by conduction. At the same time, exothermic heat is generated from chemical reaction of resins inside the composite Since the laminate is homogeneous, the distribution of temperature is expected to be symmetrical. Figure 2 shows the temperature profiles. It was observed that maximum temperature occurs on the outside and the minimum temperature is at its centre. This may be due to low thermal conductivity. The result agrees with the experimental finding that the boundary temperature is larger than the temperature inside the laminate.

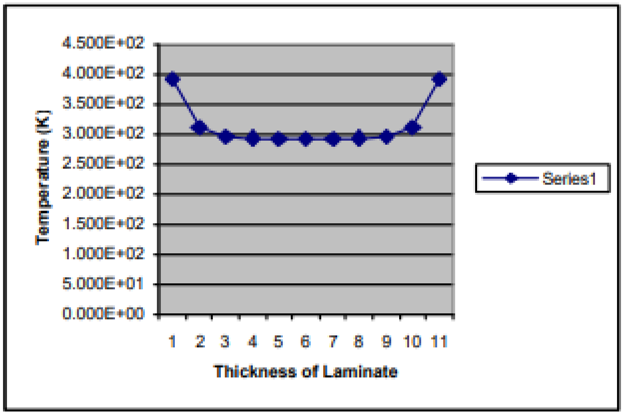

Figure 2 Temperature Profiles (time 50 miniutes)

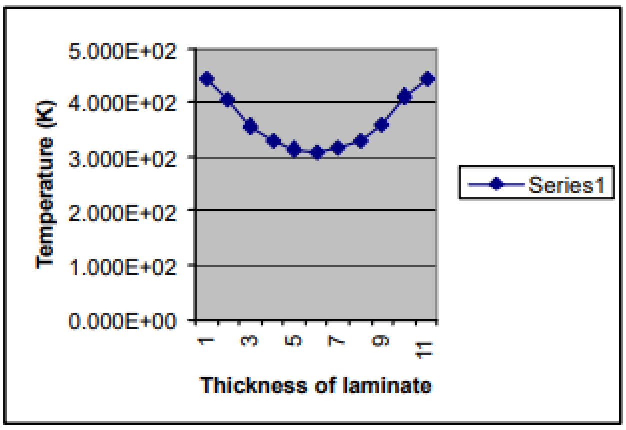

Figure 3 Temperature Profiles (time 150)

4. CONCLUSION

The AGE one-dimensional iterative method proves to be viable in determining the temperature profiles of the composite. Further study will be on determining the temperature profiles at three points in the laminate, ie, the bottom, the middle and the top, as well as the profiles of degree of cure at the center section. As temperature increases, it is expected that the exothermic reaction inside the composite may cause faster cure, and the temperature at the center to increase and become higher than the autoclave temperature.

REFERENCES

[1] C. Chepa Steven and Raymond P. Canale, Numerical Methods for Engineers with software and programming applications, 4th Ed., Mc Graw hill Boston, 2002. p.681-725.
[2] D.C. Blest, B.R. Duffy, S. McKee and A.K. Zulkifle, "Curing Simulation of Thermoset Composites", Journal: Composites: Part A ,1999, p. 1289- 1309.
[3] D.C.Blest, S. McKee, A.K. Zulkifle and P. Marshall, "Curing simulation by autoclave resin infusion", Journal: Composites Science and Techno logy 59(1999), p. 2297-2313
[4] D.J. Evans and Sahimi, M.S., "The alternating group explicit iterative method (AGE) to solve parabolic and hyperbolic partial differential equations", in Tien,C.L. and Chawla, T.C. (Ed.), Annual Review of Numerical Fluid Mechanics and Heat Transfer, United States of America , 2(1989), p. 283-389 .
[5] Zhan-Sheng Guo, Shanyi Du and Boming Zhang, "Temperature field of thick thermoset composite laminates during cure process", Journal: Composite Science and Technology 65 (2005), p.517-523.